

SPIN-UP/SPIN-DOWN OF MAGNETIZED STARS WITH ACCRETION DISKS AND OUTFLOWS

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ABSTRACT

An investigation is made of disk accretion of matter onto a rotating star with an aligned dipole magnetic field. A new aspect of this work is that when the angular velocity of the star and disk differ substantially we argue that the \mathbf{B} field linking the star and disk rapidly inflates to give regions of open field lines extending from the polar caps of the star and from the disk. The open field line region of the disk leads to the possibility of magnetically driven outflows. An analysis is made of the outflows and their back affect on the disk structure assuming an “ α ” turbulent viscosity model for the disk and a magnetic diffusivity comparable to this viscosity. The outflows are found to extend over a range of radial distances inward to a distance close to r_{to} , which is the distance of the maximum of the angular rotation rate of the disk. We find that r_{to} depends on the star’s magnetic moment, the accretion rate, and the disk’s magnetic diffusivity. The outflow regime is accompanied in general by a spin-up of the rotation rate of the star. When r_{to} exceeds the star’s corotation radius $r_{cr} = (GM/\omega_*^2)^{\frac{1}{3}}$, we argue that outflow solutions do not occur, but instead that “magnetic braking” of the star by the disk due to field-line twisting occurs in the vicinity of r_{cr} . The magnetic braking solutions can give spin-up or spin-down (or no spin change) of the star depending mainly on the star’s magnetic moment and the mass accretion rate. For a system with r_{to} comparable to r_{cr} , bimodal behavior is possible where extraneous perturbations (for example, intermittency of α , \mathbf{B} field flux introduced from the companion star, or variations in the mass accretion rate) cause the system to flip between spin-up (with outflows, $r_{to} < r_{cr}$) and spin-down (or spin-up) (with no outflows, $r_{to} > r_{cr}$).

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I. INTRODUCTION

The problem of matter accretion on to magnetized stars has been of continued interest since the discovery of X-ray pulsars in binary systems (Giacconi et al. 1971; Schreier et al. 1972; Tananbaum et al. 1972). The X-ray pulsars were interpreted as magnetized neutron stars, accreting matter from a companion star (Pringle and Rees 1972; Davidson and Ostriker 1973; Lamb, Pethick, and Pines 1973; Rappaport and Joss 1977). Of the more than 40 of known X-ray pulsars, most are in high-mass ($M_{tot} > 15$) binary systems, while only few are in low-mass ($M_{tot} < 3$) binary systems (see, for example, review by Nagase 1989).

From the early observations, many X-ray pulsars were found to show secular decreases of pulse period (spin-up of the neutron star rotation) with relatively large rates of change of $-\dot{P}/P = 10^{-2} - 10^{-6} \text{ yr}^{-1}$ (Gursky and Schreier 1975; Schreier and Fabbiano 1976; Rappaport and Joss 1983). The long-term monitoring of X-ray pulsars during the last two decades have shown a wide variety of pulse-period evolution on different time-scales. Some X-ray pulsars show a steady spin-up evolution during many years which rapidly changes to a steady spin-down evolution (e.g., GX 1+4, SMC X-1, 4U 1626-67). Others show the wavy variations of P on time-scales of a few years on a background of systematic spin-up or spin-down (e.g., Cen X-3, Her X-1, Vela X-1) (Nagase 1989; Sheffer et al. 1992; Bildsten 1993).

Short-term fluctuations of pulse-period on time-scales of days to months, including clear evidence of spin-down episodes, were found for a number of sources, for example, Her X-1 (Giacconi 1974), Cen X-3 (Fabbiano and Schreier 1977), Vela X-1 (Nagase et al. 1984). Detailed observations of Vela X-1 with Hakucho and Tenma satellites, and with HEAO-1 have revealed fluctuations of large amplitude with time-scales about 3-10 days, which were estimated to be random in both amplitude and sign (Nagase 1981). BATSE observations have shown that in some cases (Cen X-3, Her X-1) the timescale of fluctuations is less than one day with the change of sign from spin-up to spin-down and vice-versa (Bildsten 1993). Note that some X-ray pulsars (usually the long-period ones) show no evidence of regular spin-up or spin-down (for example, 4U 0115+63, GX 301-2).

In some cases, like Her X-1, the existence of an accretion disk is indicated from various

observations in the X-ray and optical bands (for example, Middleditch and Nelson 1976; Bisnovatyi-Kogan et al. 1977; Middleditch, Puetter, and Pennypacker 1985). In several other cases, such as Cen X-3, SMC X-1, GX 1+4, and 4U 1626-67, there are also different kinds of evidence, that matter from companion star forms a disk (for example, Tjemkes et al. 1986; Nagase 1989). Most of these are short-period pulsars with period about several seconds or less (excluding GX 1+4 with period 122 s.). However, most of long-period pulsars, are supposed to be powered by the capture of stellar wind from companion. However if the companion is the star of Be type with slow rate of outflow, then it is probable that once again the disk first forms, opposite to the case when the companion star is of O-B kind with high speed of matter outflow (see review of Nagase 1989). Illustrative data for two pulsars are shown in Figure 1.

Some aspects of matter accretion by magnetized neutron stars were considered for the first time by Bisnovatyi-Kogan and Friedman (1970), and by Shvartsman (1971). Ideas on the spinning-up of pulsars by accretion were first proposed in the case of disk-fed pulsars by Pringle and Rees (1972), Lamb, Pethick and Pines (1973), and Lynden-Bell and Pringle (1974), and for wind-fed pulsars by Davidson and Ostriker (1973), and Illarionov and Sunyaev (1975) (see also Lipunov 1993). The spin-down of a rapidly rotating magnetized star due to “propellar” action of the magnetosphere was suggested by Shvartsman (1971) and Illarionov and Sunyaev (1975), but a detailed model has not been worked out. Pringle and Rees (1972) considered matter accretion from a viscous disk to a magnetized rotating neutron star. Their theory considered the idealized case of an aligned rotator where the magnetic moment of pulsar is aligned parallel or anti-parallel with the pulsar’s spin-axis and the normal to the plane of the accretion disk. They supposed that accreting matter stops at the point where the magnetic pressure of the magnetosphere becomes equal to pressure of matter. At this distance, matter transfers its angular momentum to the star through a thin boundary layer. Another idea regarding the width of the transition zone between the unperturbed accretion disk and magnetosphere, was considered in series of papers by Ghosh and Lamb (1978, 1979a,b) (hereafter denoted GL). They assumed that magnetic field of the neutron star threads the accretion disk at different radii in a broad “transition” zone, in spite of predominance of the disk stress compared with magnetic

stress in this region. A schematic diagram of their model is shown in Figure 2. Their work is based on the idea of anomalous resistivity connected with the supposed fast reconnection of toroidal component of the magnetic field lines across the disk.

Here, we propose an essentially different picture for disk accretion onto a star with an aligned dipole magnetic field. The idea is based on the fact that when there is a large difference between the angular velocity of the star and that of the disk, the magnetic field lines threading the star and the disk undergo a rapid inflation so that the field becomes open with separate regions of field lines extending outward from both the star and the disk. As a result the magnetosphere consists of an open field line region far from the star and a closed region approximately corotating close to the star. This is shown schematically in Figure 3. Our model does not assume an anomalous resistivity, but rather a turbulent magnetic diffusivity of the disk comparable to the turbulent α viscosity proposed by Shakura (1973). This corresponds to a resistivity about 10^4 times smaller than that assumed by Ghosh and Lamb (1978,1979a,b). Campbell (1992) has earlier discussed closed field line models of disk accretion onto an aligned dipole star assuming a magnetic diffusivity comparable to the turbulent α viscosity.

Here, we study the matter flow in the disk taking into account the open magnetic field line region of the magnetosphere. The existence of an open magnetic field region of the disk leads to the possibility of magnetically driven outflows. We analyze the magnetohydrodynamic (MHD) outflows and their back influence on the disk using the work on MHD outflows and magnetized disks by Lovelace, Berk, and Contopoulos (1991) (denoted LBC), Lovelace, Romanova, and Contopoulos (1993) (LRC), and Lovelace, Romanova, and Newman (1994) (LRN).

In §II we discuss the basic equations. In §IIA we argue that part of the star/disk field configuration is open. In §IIB we give the basic equations for a magnetized viscous accretion disk. In §IIC we discuss magnetically driven outflows and in §IID, necessary condition for these outflows. In §IIE, we give the main results for the case of outflows where in general star spins up. In §IIF, we discuss the magnetic braking of the star by the disk due to field line twisting, which occurs when there are no outflows, and show that the star can either spin-down or spin up depending on the mass accretion rate and the

magnetic moment of the star. In §III we discuss the conclusions of this work.

II. THEORY

The basic equations for an assumed stationary configuration of plasma are:

$$\begin{aligned} \nabla \cdot (\rho \mathbf{v}) &= 0, \quad \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = 0, \\ \mathbf{J} &= \sigma_e (\mathbf{E} + \mathbf{v} \times \mathbf{B}/c), \quad \rho(\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \rho \mathbf{g} + \frac{1}{c} \mathbf{J} \times \mathbf{B} + \mathbf{F}^{\text{vis}}. \end{aligned} \quad (1)$$

Here, \mathbf{v} is the flow velocity, ρ is the density, σ_e is the effective electrical conductivity, \mathbf{F}^{vis} is the viscous force density, $p = \rho k_B T/m$ is the gas pressure (with k_B the Boltzmann constant, m the mean particle mass, and with the radiation pressure assumed negligible), and \mathbf{g} is the gravitational acceleration. Outside of the disk, dissipative effects are considered to be negligible ($\sigma_e \rightarrow \infty$, $\mathbf{F}^{\text{vis}} = 0$, etc.). We neglect the self-gravity of the disk and relativistic effects so that $\mathbf{g} = -\nabla \Phi_g$ with $\Phi_g = -GM/(r^2 + z^2)^{1/2}$, where M is the mass of the central star. Equations (1) are supplemented later by a equation for the conservation of energy in the disk.

A general axisymmetric \mathbf{B} -field can be written as $\mathbf{B} = \mathbf{B}_p + \hat{\phi} B_\phi$, where $\mathbf{B}_p = \nabla \times (\hat{\phi} \Psi/r)$ is the poloidal field, B_ϕ is the toroidal field, and Ψ is the flux function (see, for example, Mestel 1968, or Lovelace et al. 1986). We use a non-rotating cylindrical coordinate system so that $\mathbf{B}_p = (B_r, 0, B_z)$. Notice that $\Psi(r, z) = \text{const.}$ labels a poloidal field line, $(\mathbf{B}_p \cdot \nabla) \Psi = 0$, or a flux-surface if the poloidal field line is rotated about the z -axis. For the present problem, the \mathbf{B}_p field can be represented as $\Psi = \Psi_* + \Psi'$, with Ψ_* the star's field-assumed to be a dipole $\Psi_* = \mu r^2 (r^2 + z^2)^{-3/2}$ and with Ψ' due to the non-stellar toroidal currents.

A. Inflation and Opening of Coronal B-Field

Figure 4 shows a sketch of the poloidal projections of two nearby field lines connecting the star and the disk. Because $\partial/\partial t = 0$, the \mathbf{E} field is electrostatic and the poloidal plane line integral of \mathbf{E} around the loop, $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$, is zero. Because of axisymmetry and the fact that $\mathbf{E} + \mathbf{v} \times \mathbf{B}/c = 0$ outside of the disk, the line integrals of \mathbf{E} along the two curved segments in Figure 2 vanish separately. That is, the electrostatic potential is a constant on any given flux surface (see, for example, Lovelace et al. 1986). The potential difference between the points (1,2) on the star's surface is $-\delta \mathbf{r}_{12} \cdot \mathbf{E}_* = \omega_* \delta \Psi/c$,

with $\delta\Psi \equiv \Psi_1 - \Psi_2$, where we have assumed that the star is perfectly conducting [$\mathbf{E}_* = -(\mathbf{v} \times \mathbf{B})_*/c$], and where ω_* is the angular rotation rate of the star. Thus, we have $\delta r_{34}(E_r)_d = -\omega_*\delta\Psi/c = -\omega_*rB_z(r, 0)/c$, where the path element δr_{34} is in the mid plane of the disk, and the "d" subscript indicates that the quantity is evaluated in the disk. In turn, we have $E_r|_d = (J_r/\sigma_e - v_\phi B_z/c)_d$. From Ampère's law, $J_r = -(c/4\pi)(\partial B_\phi/\partial z)$, the fact that B_ϕ is necessarily an odd function of z , and the approximation $\partial B_\phi/\partial z = (B_\phi)_h/h$, we have $J_r \approx -(c/4\pi)(B_\phi)_h/h$, where h is the half-thickness of the disk, and the notation $(\dots)_h$ indicates that the quantity is evaluated at $z = h$. Combining these results gives for stationary conditions

$$(B_\phi)_h(r) = -\frac{hr}{\eta_t}[\omega_d(r) - \omega_*]B_z(r) , \quad (2)$$

where $\eta_t = c^2/(4\pi\sigma_e)$ is the magnetic diffusivity of the disk, and ω_d is the angular rotation rate of the disk. The fractional variation in B_z between the midplane and surface of the disk is negligible [$O(h/r)$] if the disk is thin in the sense that $h/r \ll 1$. For the conditions of interest here, we show in §IIB that $h/r \leq c_s/v_K$, where $c_s \equiv (k_B T/m)^{1/2}$ the Newtonian sound speed based on the mid-plane temperature of the disk, and $v_K = (GM/r)^{1/2}$ is the Keplerian velocity at r . Campbell (1992) has independently derived equation (2) but does not place any limit on the ratio $|B_\phi/B_z|$ (see discussion following equation 3).

In contrast with the work of Ghosh and Lamb (1978, 1979a,b), we assume that the magnetic diffusivity of the disk η_t is of the order of magnitude of the disk's effective viscosity (see Bisnovaty-Kogan and Ruzmaikin 1976; Parker 1979; Campbell 1992). Additionally, we assume that the disk viscosity is the turbulent viscosity as formulated by Shakura (1973) and Shakura and Sunyaev (1973), $\nu_t = (2/3)\alpha c_s h$, where α is a dimensionless quantity less than unity. In this work, we assume that α is in the range 10^{-1} to 10^{-2} . The possible contribution to the turbulent momentum flux due to small-scale magnetic field fluctuations is assumed included in α (Eardley and Lightman 1975; Coroniti 1981; Balbus and Hawley 1992; Kaisig, Tajima, and Lovelace 1992). We write $\eta_t = D\nu_t$ where D is dimensionless and of order unity.

With η_t the turbulent diffusivity, equation (2) gives $|(B_\phi)_h/B_z| = (3/2)r|\omega_d(r) - \omega_*|/(\alpha D c_s)$. This ratio is a measure of the twist of the \mathbf{B} field between the star and the disk. For the outer part of the disk $\omega_d(r)$ is expected to be close to the Keplerian value

$\omega_K = (GM/r^3)^{\frac{1}{2}}$. Thus, it would appear that the twist $|(B_\phi)_h/B_z|$ can be very much larger than unity, say, > 10 . We argue here that such large values of the twist do not occur.

For a discussion of the limitation on the twist $|(B_\phi)_h/B_z|$ we consider that the plasma outside of the disk is force-free, $\mathbf{J} \times \mathbf{B} = 0$, which is the coronal plasma limit of Gold and Hoyle (1960). This limit is valid under conditions where the kinetic energy density of the plasma is much less than $\mathbf{B}^2/8\pi$. The general response of force-free coronal magnetic field loops to stress (differential twisting) applied to the loop foot points (in the solar photosphere) has been studied intensely by Aly (1984, 1991), Sturrock (1991), and Porter, Klimchuk, and Sturrock (1992). The conclusion of these studies is that a closed field loop with a small twist evolves into an open field-line configuration as the twist is increased. The related problem of the twisting of a force free magnetic field configuration with foot points at different radii in a Keplerian disk has been studied by Newman, Newman and Lovelace (1992), and Lynden-Bell and Boily (1994). The twisting due to the differential rotation of the disk acts to increase the total magnetic energy of the coronal field and this in turn acts to “inflate” the field as shown in Figure 5.

An alternative measure of the field twist between the star and the disk is simply the difference in the azimuthal location, $\Delta\phi$, of the stellar and disk foot points of a given flux tube. We have $r d\phi/dl_p = B_\phi(r, z)/|\mathbf{B}_p(r, z)|$, where dl_p is the length element along a poloidal field line $\Psi(r, z) = \text{const}$. Because $rB_\phi(r, z)$ depends only on Ψ in a force-free plasma, we have

$$\Delta\phi = [r(B_\phi)_h]_{disk} \int_{star}^{disk} \frac{dl_p}{r|\nabla\Psi|} . \quad (3)$$

For small $\Delta\phi$ (say, < 1), the poloidal field is close to that of a dipole, and the integral gives $\Delta\phi = C_t[(B_\phi)_h/B_z]_d$ with $C_t = 8/15$. For increasing $\Delta\phi$, the value of C_t increases because of the longer path length and the smaller value of $|\nabla\Psi|$. The present situation is analogous to a current-carrying plasma column with a longitudinal B field. For the plasma column there is the well-known Kruskal-Shafranov stability condition (see, for example, Bateman 1980) on the twist $\Delta\phi$ of the field (at the column’s surface) over the length of the column. This condition, $\Delta\phi < 2\pi$, is required for stability against symmetry changing kink perturbations. A related, non-axisymmetric instability may accompany the above

mentioned inflation and opening of the star/disk field in response to increasing $\Delta\phi$.

We propose that there is a definite upper limit on the twist $\Delta\phi$ of any field lines connecting the star and the disk. This limit implies a corresponding limit on $|(B_\phi)_h/B_z|_d$ in equation (2). If $\Delta\phi$ is larger than this limit, then the field is assumed to be open as indicated in Figure (3). Note that when the field threading the disk is open equation (2) does not apply.

B. Basic Equations for the Disk

The flow velocity in the disk is $\mathbf{v}(r) = -u(r)\hat{r} + v_\phi(r)\hat{\phi}$, where u is the accretion speed. The surface density is $\sigma = \int_{-h}^h dz \rho(r, z)$. The main disk equations are:

1. Mass conservation:

$$\dot{M} = 2\pi r \sigma u, = \text{const.} \quad (4)$$

We assume that the change of \dot{M} with r due to possible MHD outflows is small.

2. Radial force balance:

$$\frac{\sigma v_\phi^2}{r} = \frac{GM\sigma}{r^2} - \frac{(B_r B_z)_h}{2\pi} + \frac{d}{dr} \int_{-h}^h dz p \quad . \quad (5)$$

3. Angular momentum conservation:

$$\begin{aligned} \frac{d}{dr}(\dot{M}F) &= -r^2(B_\phi B_z)_h, \\ F &\equiv r^2\omega + \frac{r^2\nu_t}{u} \frac{d\omega}{dr} \quad , \end{aligned} \quad (6)$$

where $\omega = v_\phi/r$. The term $-r^2(B_\phi B_z)_h$ represents the outflow of angular momentum from the $\pm z$ surfaces of the disk (that is, a torque on the disk). We assume that the angular momentum carried by the matter of the outflow is small compared with that carried by the field. If the B field at r is open as discussed in §IIA, then the angular momentum outflow from the disk is carried to infinity. On the other hand, if the field is closed, then the angular momentum outflow from (or inflow to) the disk is carried by the coronal B field to (or from) the star.

4. Energy conservation:

$$\sigma\nu_t \left(r \frac{d\omega}{dr} \right)^2 + \frac{4\pi}{c^2} \int_{-h}^h dz \eta_t \mathbf{J}^2 = \frac{4acT^4}{3\kappa\sigma} \equiv 2\sigma_B T_{eff}^4 \quad . \quad (7)$$

The first term is the viscous dissipation, the second is the Ohmic dissipation, while the third term is the power per unit area carried off by radiation (in the $\pm z$ directions) from the disk which is assumed optically thick. Here, κ is the opacity assumed due to electron scattering, a and $\sigma_B = ac/4$ are the usual radiation constants, T is the midplane temperature of the disk, and T_{eff} is disk's effective surface temperature.

5. Conservation of magnetic flux for a general time-dependent disk:

$$\frac{\partial}{\partial t}(rB_z) = \frac{\partial}{\partial r} \left[rB_z u - \frac{\eta_t r}{h} (B_r)_h \right], \quad (8a)$$

where the generally small radial diffusion of the B_z field has been neglected (see LRN). The first term inside the square brackets represents the advection of the field while the second term represents the diffusive drift. In a stationary state,

$$\beta_r(r) \equiv \frac{(B_r)_h}{B_z} = \frac{uh}{\eta_t}. \quad (8b)$$

6. Vertical hydrostatic equilibrium:

The z -component of the Navier-Stokes equation (1) gives the condition for vertical hydrostatic balance which can be written as

$$\left(\frac{h}{r}\right)^2 + b \left(\frac{h}{r}\right) - \left(\frac{c_s}{v_K}\right)^2 = 0, \quad (9)$$

where c_s is the Newtonian sound speed based on the midplane temperature of the disk, and $b \equiv r \left[(B_r)_h^2 + (B_\phi)_h^2 \right] / (4\pi\sigma v_K^2)$ (Wang, Lovelace, and Sulkanen 1990). Radiation pressure is negligible for the conditions of interest. For $b \ll 2c_s/v_K$, this equation gives the well-known relation $h/r = c_s/v_K$ (Shakura and Sunyaev 1973), while for $b \gg 2c_s/v_K$ it gives $h/r = b^{-1}(c_s/v_K)^2$ which is smaller than c_s/v_K owing to the compressive effect of the magnetic field external to the disk (Wang et al. 1990). It is useful to write $b = \epsilon \beta^2$, where $\epsilon \equiv rB_z^2(r, 0)/(2\pi\sigma v_K^2)$ and $\beta^2 \equiv [(B_r)_h^2 + (B_\phi)_h^2]/(2B_z^2)$. In §IID, we discuss that $\beta = O(1)$ in order to have outflows from the disk, and $\beta \leq 1$ with no outflows. Thus, the Alfvén speed in the midplane of the disk is $v_A = v_K \epsilon^{\frac{1}{2}} (h/r)^{\frac{1}{2}}$. We may term the magnetic field as weak if $\epsilon < c_s/v_K \approx h/r$. In this limit, $v_A/c_s \approx (\epsilon r/h)^{\frac{1}{2}} \leq 1$, and the magnetic compression of the disk is always

small. In the opposite, strong field limit, $\epsilon > c_s/v_K$. If at the same time $\beta = O(1)$, then the disk is magnetically compressed, and for $\epsilon \gg c_s/v_K$, $v_A/c_s = 1/\beta$.

The following subsections involve applications of equations (4) – (8). Note that with the magnetic field terms neglected these equations give the Shakura-Sunyaev (1973) solution for region “b”.

C. Magnetically Driven Outflows

Consider the outer region of the disk where the value of the field twist $\Delta\phi$ given by equation (3) [or $|(B_\phi)_h/B_z|$ of equation (2)] is larger than a critical value. In this region the B field threading the disk is open as discussed in §IIA. The angular momentum flux carried by the disk is $\dot{M}F$. If the angular rotation of the disk is approximately Keplerian, $\omega(r) \approx \omega_K(r)$, then the viscous transport contribution to F is $r^2(\nu/u)(d\omega/dr) \approx -(3/2)r^2\omega[h/(\beta_r r)]$. As discussed in detail below a necessary condition for MHD outflows is that $\beta_r = O(1)$. Consequently, the viscous contribution to F in equation (6) is smaller than the bulk transport term by the small factor $h/r \ll 1$, and $F \approx \omega r^2$.

In equation (6) we therefore have $d(\dot{M}F)/dr \approx \dot{M}\omega_K r/2$. In equation (6) we let $(B_\phi)_h = -\beta_\phi B_z(r, 0)$. Studies of MHD outflows (Blandford and Payne 1982, LBC, LRC) indicate that $\beta_\phi = \text{const.} \lesssim O(1)$. As a result, equation (6) implies that

$$[B_z(r, 0)]_w = k/r^{5/4}, \text{ with } k = [\dot{M}(GM)^{1/2}/(2\beta_\phi)]^{1/2}, \quad (10)$$

where the w subscript indicates the wind region of the disk. Of course at a sufficiently small radius, denoted r_{wi} , the outflow ceases, and the dependence of $B_z(r, 0)$ reverts to approximately the stellar dipole field, μ/r^3 . As a consistency condition we must have

$$\mathcal{E} = (k/r_{wi}^{5/4})/(\mu/r_{wi}^3) = \left[\frac{\dot{M}(GM)^{1/2}r_{wi}^{7/2}}{\mu^2(2\beta_\phi)} \right]^{1/2} < 1. \quad (11)$$

The transition between the dipole and outflow field dependences is handled by letting $B_z(r, 0) = (\mu/r^3)g(r) + (k/r^{5/4})[1 - g(r)]$, where, for example, $g(r) = \{1 + \exp[(r - r_{wi})/\Delta r]\}^{-1}$ and $\Delta r/r_{wi} \ll 1$.

Equations (4) – (9) can be used to obtain the disk parameters in the region of outflow. As shown below the viscous dissipation is much larger than the Ohmic, and equation (7)

then gives $T = [81\dot{M}^2(GM)^{1/2}\kappa/(128\pi^2\alpha D\beta_r^2 ac)]^{1/4}r^{-7/8}$. For illustrative values, $M = 1M_\odot$, $\dot{M} = 10^{17}\text{g/s} \approx 1.6 \times 10^{-9}M_\odot/\text{yr}$, $\alpha = 0.1$, $\beta_r = 0.58$, and $D = 1$, we find at the representative distance $r = 10^8\text{cm}$ that $T \approx 0.44 \times 10^6 K$, $v_K = 1.15 \times 10^9\text{cm/s}$, $c_s = 7.6 \times 10^6\text{cm/s}$, $u = 0.88 \times 10^6\text{cm/s}$, $\sigma = 180\text{g/cm}^2$, and $h/r = 6.6 \times 10^{-3}$. The ratio of the ohmic dissipation to that due to viscosity is: $(4/27)\alpha D\beta_r^3 \ll 1$. The ratio of the viscous dissipation to the power output of the outflows (the $\pm z$ Poynting fluxes) (per unit area of the disk) is: $(9/2)(h/r)(\beta_r D)^{-1} \ll 1$. Thus, most of the accretion power for $r > r_{wi}$ goes into the outflows. However, the fraction of the total accretion power in outflows is small if $r_{wi} \gg r_*$, where r_* is the star's radius ($\sim 10^6\text{cm}$ for a neutron star). Note that $\dot{M} = 10^{17}\text{g/s}$ and $M = 1M_\odot$ correspond to a total accretion power or luminosity $L_0 = GM\dot{M}/r_* \approx 1.33 \times 10^{37}(\text{erg/s})(10^6\text{cm}/r_*)$.

Conservation of the poloidal flux threading the disk implies that there is an outer radius, denoted r_{wo} , of the outflow from the disk. That is, we have $\int_{r_{wi}}^{r_{wo}} r dr (k/r^{5/4}) = \int_{r_{wi}}^{r_{wo}} r dr (\mu/r^3)$. Thus, the outer radius is $r_{wo} \approx r_{wi}(f\mathcal{E})^{-4/3}$, where $f \approx 1$ for $1 - \mathcal{E} \ll 1$ and $f = 3/4$ for $\mathcal{E} \ll 1$. For $r > r_{wo}$, $B_z(r, 0)$ rapidly approaches zero.

Equation (9) gives the disk thickness. From equation (10), we have $\epsilon = rB_z^2/(2\pi\sigma v_K^2) = (u/c_s)(c_s/v_K)/(2\beta_\phi)$. The magnetic compression of the disk is small if $\epsilon < c_s/v_K$ or equivalently if $u/c_s < 1$. Our numerical solutions have $u/c_s < 1$ in the region of outflow.

D. Necessary Condition for MHD Outflows

A necessary condition on β_r for MHD outflows from the disk can be obtained by considering the net force \mathcal{F}_p on a fluid particle in the direction of its poloidal motion above the disk. Notice that for stationary flows, the poloidal flow velocity \mathbf{v}_p is parallel to \mathbf{B}_p owing to the assumed axisymmetry and perfect conductivity. However, in general $\mathbf{v} \times \mathbf{B} \neq 0$ so that the fluid particles do not move like “beads on a wire”. Above, but close to the disk ($h \leq z \ll r$), we assume that the poloidal field lines are approximately straight. Thus, the poloidal position of a fluid particle above the disk is $\mathbf{r} = (r_o + S\sin(\theta))\hat{\mathbf{r}} + S\cos(\theta)\hat{\mathbf{z}}$, where S is the distance along the path from the starting value $S_o = h/\cos(\theta)$ at a radius r_o , $\tan(\theta) = (B_r)_h/B_z = \beta_r$, and $z = S\cos(\theta)$. The effective potential is $U(S) = -(1/2)\omega_o^2(r_o + S\sin(\theta))^2 - GM/|\mathbf{r}|$ (LRN), and the force in the direction of the particle's poloidal motion is $\mathcal{F}_p = -\partial U/\partial S$. For distances z much less than the distance

to the Alfvén point, $\omega_o \approx \omega_K(r_o)$. In this way we find

$$\mathcal{F}_p = -(\omega_K^2 - \omega^2)r \left(\frac{B_r}{|\mathbf{B}_p|} \right) + \omega_K^2 z (3\beta_r^2 - 1) \left(\frac{B_z}{|\mathbf{B}_p|} \right), \quad (12)$$

for $h \lesssim z \ll r$, where we assume $B_z > 0$.

The slow magnetosonic point of the outflow occurs where $\mathcal{F}_p = 0$ (LRC) at the distance

$$z_s = r \left(1 - \frac{\omega^2}{\omega_K^2} \right) \frac{\beta_r}{3\beta_r^2 - 1}. \quad (13)$$

The factor $1 - (\omega/\omega_K)^2$ can be obtained from the radial force balance equation (5). The radial pressure force is small compared with the magnetic force for the conditions we consider, $\alpha\beta_r^2/(3\beta_\phi) > h/r \ll 1$. Thus,

$$\frac{z_s}{h} = \left(\frac{2\alpha D B_z^2 r}{3M\omega_K} \right) \frac{\beta_r^3}{3\beta_r^2 - 1}. \quad (14)$$

For $r_{wi} < r < r_{wo}$, the quantity in brackets is simply $\alpha/(3\beta_\phi)$ owing to equation (10). Note that the minimum of $\beta_r^3/(3\beta_r^2 - 1)$ is $1/2$ at $\beta_r = 1$. If the gas near the surface of the disk ($h \lesssim z \leq z_s$) is assumed isothermal with temperature $T_1 \ll T$ and sound speed $c_{s1} \equiv (k_B T_1/m)^{1/2}$, then the density at the slow magnetosonic point ρ_s can be obtained using the MHD form of Bernoulli's equation (LRC). At the slow magnetosonic point, the poloidal flow speed is $v_p = c_{s1}|\mathbf{B}_p|/|\mathbf{B}| \equiv v_{sm}$, the slow magnetosonic speed, and $\rho_s = \rho_h \exp \{-1/2 - [U(z_s) - U(h)]/c_{s1}^2\}$, where U is now regarded as a function of z . In turn, the mass flux density from the $+z$ surface of the disk is $\rho_s v_{sm} \cos(\theta) = \rho_s c_{s1} B_z/|\mathbf{B}|$.

The consistency of the magnetically driven outflow solutions requires $z_s \gtrsim h$. If this were not the case, the outflows would be matter rather than field dominated near the disk. There are two possible regimes having $z_s \gtrsim h$: one is with $\alpha D/(3\beta_\phi) < 1$ [or $\alpha D/(3\beta_\phi) \ll 1$] and $\beta_r \gtrsim 1/\sqrt{3}$; another is with $\alpha D/(6\beta_\phi) > 1$ and $\beta_r > 1$. In the present work, we consider the first regime which is consistent in the following respect: The magnetic pinching force on the inner part of the outflow ($r < r_{wo}$) increases as the mass flux in the outflow increases (LBC). In turn, an increase in the pinching force acts to decrease $\beta_r = (B_r)_h/B_z$ thereby increasing z_s/h and decreasing the mass flux in the outflow.

In addition to the condition $z_s \gtrsim h$, the slow magnetosonic point must be at a distance z_s *not* much larger than h , say $2h$, in order for the outflow to have a non-zero mass flux. At the inner radius of the outflow, r_{wi} , the $B_z(r, 0)$ field in the disk is $\mu/(2r_{wi}^3)$ as discussed in §IIC. We then have

$$r_{wi} \gtrsim \left[\frac{\alpha D \mu^2}{24 \dot{M} (GM)^{\frac{1}{2}}} \right]^{\frac{2}{7}} \approx 0.75 \times 10^8 \text{cm} \left[\left(\frac{\alpha D}{0.1} \right) \left(\frac{\mu}{10^{30} \text{Gcm}^3} \right)^2 \left(\frac{10^{17} \text{g/s}}{\dot{M}} \right) \left(\frac{M_{\odot}}{M} \right)^{\frac{1}{2}} \right]^{\frac{2}{7}}, \quad (15)$$

from equation (14). Note that from equation (11), we get $\mathcal{E} \gtrsim [\alpha D / (48 \beta_{\phi})]^{\frac{1}{2}}$ or $\mathcal{E} \gtrsim 0.046$ and $r_{wo} \lesssim 90 r_{wi}$ for $\alpha = 0.1$, $D = 1$ and $\beta_{\phi} = 1$.

E. MHD Outflows and Spin-Up ($r_{to} < r_{cr}$)

We have numerically integrated equations (4) – (9) assuming magnetically driven outflows from the disk as discussed in §§IIC & D. We integrate the equations inward starting from a large radial distance $r < r_{wo}$. The inner radius of the region of outflow is determined as indicated in §IID. For $r < r_{wi}$, $dF/dr = 0$, and the solutions in this range of r all exhibit a “turn-over radius”, r_{to} , where $d\omega/dr = 0$. This “turn over” results from the radially outward magnetic force in equation (5) which becomes stronger as r decreases. In the region close to the turnover, the magnetic compression of the disk becomes strong in that $\epsilon = O(1)$. Figure 6 shows a representative solution. From a least squares fitting of many integrations, we find

$$r_{to} \approx 0.91 \times 10^8 \text{cm} \left(\frac{\alpha D}{0.1} \right)^{0.3} \left(\frac{\mu}{10^{30} \text{Gcm}^3} \right)^{0.57} \left(\frac{10^{17} \text{g/s}}{\dot{M}} \right)^{0.3} \left(\frac{M_{\odot}}{M} \right)^{0.15}. \quad (16)$$

Figure 7 shows as an example the dependence of r_{to} on μ . The turn-over radius is less than but in all cases close to r_{wi} . Thus, equation (16) is compatible with equation (15). Our radius r_{to} has a role similar to that of the Alfvén radius r_A of Ghosh and Lamb (1978, 1979a,b). The dependences we find of r_{to} on μ , \dot{M} , and M are close to those of r_A . However, our analysis shows an important dependence on αD which is proportional to the magnetic diffusivity of the disk. Furthermore, our r_{to} is smaller than r_A by a factor of about $(\alpha D / 12)^{0.3}$, which is ≈ 0.24 for $\alpha D = 0.1$.

The turn-over radius is important in the respect that the inward angular momentum flux carried by the disk ($\dot{M}F$) is $\dot{M} \omega_{to} r_{to}^2$ because $d\omega/dr = 0$ at $r = r_{to}$. [Note that we

do not consider here the possibility of significant poloidal current flow along the open field lines extending from the polar caps of the star ($B_\phi > 0$ for $z > 0$ in figure 3) which would act to remove angular momentum from the star to infinity.] From numerical integrations, we find to a good approximation that $\omega_{to} = (GM/r_{to}^3)^{\frac{1}{2}}$, so that the influx of angular momentum to the star is $\dot{M}F_{to} = \dot{M}(GMr_{to})^{\frac{1}{2}}$. Thus, the rate of increase of the star's angular momentum J is:

$$\frac{dJ}{dt} = \dot{M}(GMr_{to})^{\frac{1}{2}}. \quad (17)$$

With I the moment of inertia of the star, $J = I\omega_*$, and $dJ/dt = (dI/dM)\dot{M}\omega_* + I(d\omega_*/dt)$. For the situation of interest, the term proportional to dI/dM is negligible because r_{to} is much larger than the star's radius. Thus, we have “spin-up” of the star,

$$\frac{d\omega_*}{dt} = \frac{\dot{M}(GMr_{to})^{\frac{1}{2}}}{I}, \quad (18a)$$

or

$$\frac{1}{P} \frac{dP}{dt} \approx -5.8 \times 10^{-5} \frac{1}{\text{yr}} \left(\frac{P}{1s} \right) \left(\frac{\dot{M}}{10^{17} \text{g/s}} \right) \left(\frac{10^{45} \text{gcm}^2}{I} \right) \left(\frac{M}{M_\odot} \right)^{\frac{1}{2}} \left(\frac{r_{to}}{10^8 \text{cm}} \right)^{\frac{1}{2}}, \quad (18b)$$

where $P = 2\pi/\omega_*$ is the pulsar period. Because the total accretion luminosity $L = GM\dot{M}/r_*$ with r_* the star's radius, $\dot{P} \propto -P^2 L^{0.85} \mu^{0.285}$ for constant $\alpha D, r_*$, and M .

Our outflow solutions are consistent only in the case where $\omega_{to} = (GM/r_{to}^3)^{\frac{1}{2}} > \omega_*$. In this case, with r decreasing from r_{to} , the rotation rate of the disk $\omega(r)$ decreases and approaches the rotation rate of the magnetosphere ($\approx \omega_*$) from above where it matches onto the magnetospheric rotation through a radially thin turbulent boundary layer. We find no consistent stationary solutions when $\omega(r)$ decreases below ω_* . The condition $\omega_{to} > \omega_*$ is the same as

$$r_{to} < r_{cr} \equiv \left(\frac{GM}{\omega_*^2} \right)^{\frac{1}{3}} \approx 1.5 \times 10^8 \text{cm} \left(\frac{M}{M_\odot} \right)^{\frac{1}{3}} \left(\frac{P}{1s} \right)^{\frac{2}{3}}, \quad (19)$$

where r_{cr} is commonly referred to as the “co-rotation radius”. Note that r_{cr} decreases during spin-up. Thus,

r_{cr} approaches r_{to} if r_{to} does not decrease rapidly.

F. Magnetic Braking of Star by Disk; Spin-Up or Spin-Down ($r_{to} > r_{cr}$)

For $r_{to} > r_{cr}$, the B field in the outer part of the disk ($r \gg r_{cr}$) is open as discussed in §IIA, but there are no consistent stationary MHD outflows, $(B_\phi)_h = 0$, $\beta_r = hu/\eta_t \ll 1$, and the Ohmic dissipation is small compared with that due to viscosity. Thus, the outer part of the disk obeys essentially the equations of Shakura and Sunyaev (1973) for a Keplerian disk with $F = \text{const.} \equiv F_\infty$ in equation (6) undetermined. We show below that F_∞ can be determined self-consistently by considering the region near r_{cr} where the Keplerian accretion flow is brought into co-rotation with the star. Because there are no outflows, the angular momentum influx to the star is $\dot{M}F_\infty$ which is equal to dJ_*/dt .

In the region of the disk where $\omega(r)$ is close to ω_* , that is, where $r \sim r_{cr}$, $|(B_\phi)_h/B_z|$ given by equation (2) is *not* much larger than unity. In this region, closed but twisted field lines link the star and the disk. The twist of the field acts to remove (or add) angular momentum from (to) the disk if $B_\phi B_z > 0$ (or < 0). The angular momentum removed from the disk is deposited on the star via the B field. In this region of the disk the key equations are (2) and (6) which we rewrite as

$$\frac{d\omega}{dr} = \frac{u}{r^2\nu_t}(F - \omega r^2) , \quad (20a)$$

$$\frac{dF}{dr} = \frac{hr}{D\nu_t}(\omega - \omega_*)\left(\frac{r^2 B_z^2}{\dot{M}}\right)H\left(\frac{\tau}{\tau_{max}}\right) . \quad (20b)$$

Here, $H(x)$ is a Heaviside function such that $H(x) = 1$ for $|x| < 1$ and $H(x) = 0$ for $|x| > 1$;

$$\tau(r) \equiv \frac{(B_\phi)_h(r)}{B_z(r, 0)} = -\frac{hr}{D\nu_t}(\omega - \omega_*) \quad (20c)$$

is a measure of the field twist; $\tau_{max} = \text{const.}$ is the maximum value of the twist; and

$$F(r) \equiv r^2\omega + \frac{r^2\nu_t}{u}\frac{d\omega}{dr} . \quad (20d)$$

We consider τ_{max} to be a universal constant. For $|\tau| > \tau_{max}$ the field configuration becomes open. In equations (18), B_z is assumed to be the star's dipole field. The other equations of §IIB are still needed. Note that both the viscous and Ohmic heating must be retained in the energy equation (7). The viscous dissipation is dominant for $r > r_{cr}$, while the Ohmic dissipation dominates for $r < r_{cr}$. Equations (5) and (8b) can be combined to give u as a function of r, T , and ω . In turn, this expression for u can be combined with the energy

conservation equation (7) to give both u and T as functions of r, ω , and $\partial\omega/\partial r$ if viscous heating dominates, or as functions of r and ω if Ohmic heating dominates. Furthermore, $\nu_t = (2/3)\alpha c_s h$ in equation (20) can be derived from T and h/r from equation (9) using β_r from equation (8b), $\beta_\phi = -\tau$, and ϵ from B_z and u .

We have solved the equations (5, 7-9, & 20) by numerical integration starting from the outside, at a distance where $\tau > \tau_{max}$ and $F = const. = F_\infty$, and integrating inward through r_{cr} . The fact that the value of F_∞ is not known a priori points to the use of a “shooting method” for its determination. Using this approach, we find that there is in general a unique value of F_∞ , denoted F_∞^0 , such that the solution for $\omega(r)$ smoothly approaches ω_* for r decreasing below r_{cr} . If F_∞ is smaller than F_∞^0 , then $\omega(r)$ follows the Keplerian law as r decreases below r_{cr} . On the other hand, if F_∞ is larger than F_∞^0 , then $\omega(r)$ goes through a maximum and decreases rapidly to values much less than ω_* . The only physical solution is that with $F_\infty = F_\infty^0$. Hereafter, the zero superscript on F_∞ is implicit.

Figure 8 shows the radial dependences of the main physical quantities for a case of magnetic braking. The behavior of equations (20) can be understood qualitatively by noting that if $\omega(r)$ were to decrease linearly through r_{cr} , then the radial width of the region of braking, $dF/dr < 0$ where $\omega < \omega_*$, would equal that the width of the region of acceleration, $dF/dr > 0$ where $\omega > \omega_*$. Consequently, F would change by only a small fractional amount. However, if within the braking region $F(r)$ attains a value equal to ωr^2 , then $d\omega/dr = 0$ at this point and $d\omega/dr$ remains small and $\omega \approx \omega_*$ for smaller r . A rough estimation gives the radial width of the braking region as $\Delta r \sim \tau_{max} D c_s / \omega_K$, where c_s is based on the temperature just outside of this region, and the jump in F as $\Delta F \sim \tau_{max} \Delta r (r^2 B_z^2 / \dot{M})$.

Figure 9 shows the dependence of F_∞ on μ and on \dot{M} for sequences of cases where r_{cr} is assumed comparable to r_{to} . This is clearly not a necessary choice; that is, r_{to} could differ significantly from r_{cr} . However, with $r_{cr} = r_{to}$, Figure 9 allows a direct comparison of the angular momentum influx to the star ($\dot{M} F_K$) for the case of outflows ($r_{to} \lesssim r_{cr}$) with the influx ($\dot{M} F_\infty$) in the case of no outflows and magnetic braking ($r_{to} > r_{cr}$). Figure 9 suggests that a bimodal behavior can occur where the star switches between spin-up and

spin-down, owing to some extraneous perturbation.

III. DISCUSSION

This work has made a new investigation of the problem of disk accretion of matter onto a rotating star with an aligned dipole magnetic field. This is a significant idealization in that the relevant objects are mis-aligned X-ray pulsars in binary systems. Nevertheless, the aligned case is important for understanding the theory and as guide to theory of the mis-aligned rotator.

The important new ideas introduced in this work are: (1) When the angular velocity of the star and disk differ substantially, the B field between the star and disk rapidly “inflates” (see Figure 5) and this gives rise to regions of open field lines extending from the polar caps of the star to infinity and from the disk to infinity as shown in Figure 3. (As in earlier models, there is a closed donut shaped magnetosphere surrounding the star where the matter rotates at about the angular rate of the star.) (2) The open field line region of the disk may have magnetically driven outflows or winds from its $\pm z$ surfaces, and these outflows can have an important back reaction on the disk structure. We find that outflows occur when the “turn over radius” r_{to} of equation (16) is less than the “co-rotation radius” $r_{cr} = (GM/\omega_*^2)^{\frac{1}{3}}$, where ω_* is the rotation rate of the star. The solutions we find with outflows from the disk give an influx of angular momentum to the star, that is, the star spins up. Thus, a system initially with $r_{to} < r_{cr}$ evolves with r_{cr} decreasing towards r_{to} if r_{to} does not decrease rapidly. A necessary condition for MHD outflows is that the slow magnetosonic point of the outflow be located relatively close to the disk surface, which is possible if the angular rotation rate of the disk ω is less than the Keplerian rate by a small fractional amount and $\beta_r = (B_r)_h/B_z = O(1)$. If ω is significantly less than the Keplerian value, MHD outflows do not occur. Under these conditions the angular momentum flux in the disk is carried mainly by the bulk transport of the accreting matter and it is inward; that is, the viscous transport is small.

In our treatment of the disk, we assume the “ α ” turbulent viscosity model of Shakura and Sunyaev (1973), and, further, that the magnetic diffusivity of the disk is of the order of the turbulent viscosity (Bisnovatyi-Kogan and Ruzmaikin 1976; Parker 1979). This magnetic diffusivity is smaller than the anomalous diffusivity invoked by Ghosh and Lamb (1978, 1979a, b) by a factor of order 10^4 . Also, in contrast with the model of Ghosh and

Lamb, accreting matter inside of r_{to} or r_{cr} (if $r_{cr} < r_{to}$) remains in a disk. Funnelling of matter onto the star’s magnetic poles may result from mis-alignment of the stellar dipole field.

When $r_{to} > r_{cr}$, we argue that outflows do not occur but rather that there is “magnetic braking” of the accretion flow in the vicinity of r_{cr} . The braking solutions we find can give spin-up or spin-down (or no spin change) of the star depending mainly on the star’s magnetic moment and the mass accretion rate. In contrast with the case of outflows, the angular momentum flux in the disk is due to both the bulk transport (inward) and the viscous transport (outward). For braking solutions with r_{cr} comparable with r_{to} , we find spin-down of the star for large values of the star’s magnetic moment and/or large values of the mass accretion rate. Thus, for a system with r_{cr} comparable to r_{to} , bimodal behavior is possible where the system flips between spin-up ($r_{to} < r_{cr}$) and spin-down. This transistion could be triggered by a number of factors, for example, intermittency of α , B -field flux introduced from the companion star, or variation in the mass accretion rate, owing to the dependence of r_{to} on α , μ , and \dot{M} . The pulsar *GX 1 + 4* shown in Figure 1 illustrates the bimodal behavior observed for some objects. Future work is planned to address the time-dependent accretion onto an aligned dipole rotator including magnetically driven outflows from the disk.

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FIGURE CAPTIONS

Figure 1. The figure shows the pulse period history for pulsars Her X-1 (a) and GX 1+4 (b). The first shows mainly spin-up behavior with $\dot{P} = -3.35 \times 10^{-6}$ s/yr. Characteristic “wavy” fluctuations of pulse period, with episodes of spin-down, $\dot{P} = 0$, are also observed. The second pulsar shows steady spin-up with $\dot{P} = -2.55$ s/yr (GINGA observations, Bildsten 1993) and then a change to spin down, $\dot{P} = +1.5$ s/yr, and later $\dot{P} = +2.51$ s/yr (BATSE observations, Bildsten 1993).

Figure 2. The figure shows the model of Ghosh and Lamb (1978). The existence of a broad transition zone with closed magnetic field lines was suggested in this paper.

Figure 3. A schematic drawing of the magnetic field configuration considered in this work. The magnetosphere consists of an inner part, where the magnetic field lines are closed and an outer part where the field lines are open.

Figure 4. The figure shows two poloidal field lines used in the derivation of equation (2).

Figure 5. The figure shows perspective views of illustrative field-line loops in the force-free corona of a Keplerian disk. In the bottom panel the difference in the angular displacement of the foot points $\Delta\phi$ is zero, while the displacement increases going to the middle panel ($\Delta\phi \approx 80^\circ$), and then to the top panel ($\Delta\phi \approx 320^\circ$). These field lines were determined using the equations of Newman, Newman and Lovelace (1992).

Figure 6. The figure shows the radial dependences of the main physical quantities for a case where there are MHD outflows from the $\pm z$ surfaces of the disk and where the pulsar spins up. In the plot of the disk’s rotation rate, r_{to} is the “turn over radius” where $d\omega/dr = 0$. In the plot of F , the radial scale is different from the other plots. In this plot r_{wi} is the inner radius of the MHD outflow. We have taken $\alpha = 0.1$, $\beta_\phi = 1$, $D = 1$,

$\mu = 10^{30} \text{Gcm}^3$, $\dot{M} = 10^{17} \text{g/s}$, and $M = M_\odot$. The values of ω and ω_K are in units of $(GM/r_o^3)^{\frac{1}{2}}$, $F_o = (GM r_o)^{\frac{1}{2}}$, where $r_o = 0.91 \times 10^8 \text{cm}$. The quantity T_{ss} is the Shakura and Sunyaev (1973) value.

Figure 7. The figure shows the dependence of the turn-over radius r_{to} on μ for a case where $\alpha = 0.1$, $\beta_\phi = 1$, $D = 1$, $\dot{M} = 10^{17} \text{g/s}$, and $M = M_\odot$.

Figure 8. The figure shows the radial dependences of the main physical quantities for a case where there is *no* outflow, but instead magnetic braking of the accretion flow. For the case shown, the influx of angular momentum $\dot{M}F_\infty > 0$, and thus the star spins up. For this plot, $\alpha = 0.1$, $D = 1$, $\tau_{max} = 5$, $\mu = 10^{30} \text{G cm}^3$, $\dot{M} = 10^{17} \text{g/s}$, $M = M_\odot$, $r_{cr} = 0.91 \times 10^8 \text{cm}$, and $F_o = (GM r_{cr})^{\frac{1}{2}}$. For these values $r_{cr} = r_{to}$. The quantity T_{ss} is the Shakura and Sunyaev (1973) value.

Figure 9. The figure shows the dependence of F_∞ and $F_K = (GM r_{cr})^{\frac{1}{2}}$ on the pulsar period P assuming $r_{cr} = r_{to}$ for $\dot{M} = 10^{17} \text{g/s}$ and $4 \times 10^{17} \text{g/s}$. The other parameters have been taken to be $\alpha = 0.1$, $D = 1$, $\tau_{max} = 5$, and $M = M_\odot$. Thus, each $\dot{M} = \text{const.}$ curve corresponds to different magnetic moments as $\mu \propto P^{1.17}$. F_∞ and F_K are measured in units of $(GM r_o)^{\frac{1}{2}}$ with $r_o = 10^8 \text{cm}$. This plot allows a comparison of the angular momentum influx to the star ($\dot{M}F_K$) in the case of outflows and spin-up ($r_{to} < r_{cr}$) with the case of magnetic braking where the influx is $\dot{M}F_\infty(r_{to} > r_{rc})$.

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